Working Document for Breakout Group on Day 4
Multi-Species, Multi-Population Model

1 Introduction

The final model combines all of the previous models allowing several sub-populations as well as several species, each of which has active recruitment. Much of the detail here has been explained in previous working documents.

As in the previous models, there are susceptible, infected and dead of infection individuals as well as freely drifting infectious particles.

This model code could be used to recover all previous model codes, with certain choices for parameters. In its current configuration, however, this model does not allow the number of sub-populations in either direction to be 1. This limitation will be removed sometime in the near future.

2 Model Equations

The following interactions are active for a given species, s and a given sub-population, $(x,\ y)$, were x and y are integers ranging from 1 to Nx or Ny, respectively. The symbol Σ_i indicates a sum over all species, where the index i ranges from 1 to $PAR.Nspecies$. The infection parameter is a table in which the first index indicates the species doing the infection and the second index is the species being infected.

$$
\begin{aligned}
\frac{d\,S(s,y,x)}{dt} \quad = \quad & -\Sigma_i\, \beta_p(i,s)\, h\, P(i,y,x)\, S(s,y,x) \\
& -\Sigma_i\, \beta_i(i,s)\, I(i,y,x)\, S(s,y,x) \\
& -\Sigma_i\, \beta_d(i,s)\, D(i,y,x)\, S(s,y,x) \\
& +max(0, [1-(S(s,y,x)+I(s,y,x))/k(s))]\, (a_s\, S(s,y,x)+a_i\, I(s,y,x)) \\
& -m_b\, S(s,y,x) \\
\frac{d\,I(s,y,x)}{dt} \quad = \quad & +\Sigma_i\, \beta_p(i,s)\, h\, P(i,y,x)\, S(s,y,x) \\
& +\Sigma_i\, \beta_i(i,s)\, I(i,y,x)\, S(s,y,x) \\
& +\Sigma_i\, \beta_d(i,s)\, D(i,y,x)\, S(s,y,x) \\
& -m_i\, I
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\,D(s,y,x)}{dt} &= m_i\,I(s,y,x) - e\,D(s,y,x) \\
\frac{d\,P(s,y,x)}{dt} &= c_i\,I(s,y,x) + c_d\,D(s,y,x) - r\,P(s,y,x)
\end{aligned}
$$

Note that the order of the arguments is dictated by the MATLAB choice of having the first index be "row" (naturally y) and the second index be "column" (naturally x). This choice above matches the indexing in the MATLAB scripts but is otherwise irrelevant.

Particle exchange is implemented with a speed parameter Uex or Vex which characterizes x directed or y directed exchange, respectively. These parameters have units of 1/s and represent a speed times a distance between subpopulations. Specifically, these parameters represent movement in or out of the right side of a cell (Uex) or the top side of a cell (Vex).

Particle exchange is implemented by calculating $NUex = .5 * (abs(Uex) - Uex)$ which is nonzero only if the exchange parameter is negative, and $PUex = .5 * (abs(Uex) + Uex)$ which is nonzero only if the exchange parameter is positive. Similar expressions exist for $PVex$ and $NVex$.

For any given cell, only one of these two parameters is non-zero. Then $PUex(y,x)$ representes movement from cell (y,x) to cell $(y,x{+}1)$; $NUex(y,x)$ represents movement in the opposite direction. Similarly, $PVex(y,x)$ represents movement from cell (y,x) to cell $(y+1,x)$; $PVex(y,x)$ represents movement in the opposite direction.

The following terms in the equations calculate the appropriate particle exchanges.

$$
\begin{aligned}
\frac{d\,P(s,y,x)}{dt} = \; & -PUex(y,x)\,P(s,y,x) \\
& +NUex(y,x)\,P(s,y,x+1) \\
& +PUex(y,x-1)\,P(s,y,x-1) \\
& -NUex(y,x-1)\,P(s,y,x) \\
& -PVex(y,x)\,P(s,y,x) \\
& +NVex(y,x)\,P(s,y+1,x) \\
& +PVex(y-1,x)\,P(s,y-1,x) \\
& -NVex(y-1,x)\,P(s,y,x) \\
& +\text{other terms}
\end{aligned}
$$