The generation of waves by the wind

- **how does the wind generate waves?**
- **how do waves grow from a calm surface to a fully developed sea?**
- **what factors affect the characteristics of the wind-waves?**
- **how is energy being transferred from the atmosphere to the ocean?**
- **can we predict the waves if we know the wind?**

→These are very complex processes to understand and formulate, since they involve:

> **- turbulent air flow over the moving ocean surface and the interaction between the two mediums…**

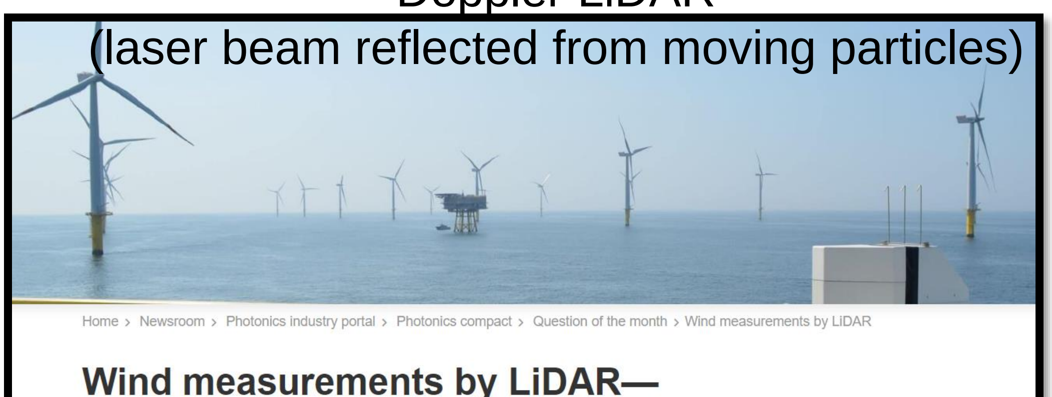
(from UK MET Office)

How we measure wind

Cup Anemometer
(speed & direction at 10m)

Sonic Anemometer

Doppler LiDAR
(laser beam reflected from moving particles)



• The **Beaufort Scale (1805…)** of wind speed is based on observations and can help to estimate wind speed. However, this estimate is only valid if sufficient time of wind blowing for fully developed sea and waves are generated only within a local weather system

Table 10.3 The Beaufort Scale

Beaufort Number	Wind Speed: Kilometers Per Hour	Wind Speed: Miles Per Hour	Effects Observed at Sea	Effects Observed on Land
0	<1	<1	Sea like a mirror	Calm; smoke rises vertically
1	1–5	1–3	Ripples with appearance of scales (capillary waves), no foam	Smoke drifts to indicate wind direction; weather vanes don't move
2	6–11	4–7	Small wavelets, crest glassy and not breaking	Wind felt on face; leaves rustle; weather vanes begin to move
3	12–19	8–12	Wave crests begin to break; scattered whitecaps	Leaves in constant motion; lightweight flags extended
4	20–28	13–18	Small waves of longer wavelengths; numerous whitecaps	Dust, leaves, loose paper raised up; small branches move
5	29–38	19–24	Moderate waves, many whitecaps, some spray	Small trees begin to sway
6	39–49	25–31	Larger waves forming whitecaps everywhere, more spray	Large trees in motion; whistling heard in wires
7	50–61	32–38	Sea heaps up; white foam from breaking waves begins to be blown in streaks	Large trees in motion; resistance felt when walking against wind
8	62–74	39–46	Moderately high waves of long wavelength; foam is blown from crests in well-marked streaks	Small branches break off trees; walking progress impeded
9	75–88	47–54	High waves, dense streaks of foam, spray may reduce visibility	Slight structural damage; shingles are blown from roofs
10	89–102	55–63	High waves with overhanging crests; sea takes on white appearance from foam; visibility reduced	Trees broken or uprooted; considerable structural damage
11	103–117	64–72	Exceptionally high waves; sea covered with white foam; visibility sporadic	Very rarely experienced on land; usually accompanied by widespread damage
12	>118	>73	Hurricane conditions; air filled with foam; sea white with driven spray; visibility greatly reduced	Catastrophic damage to structures

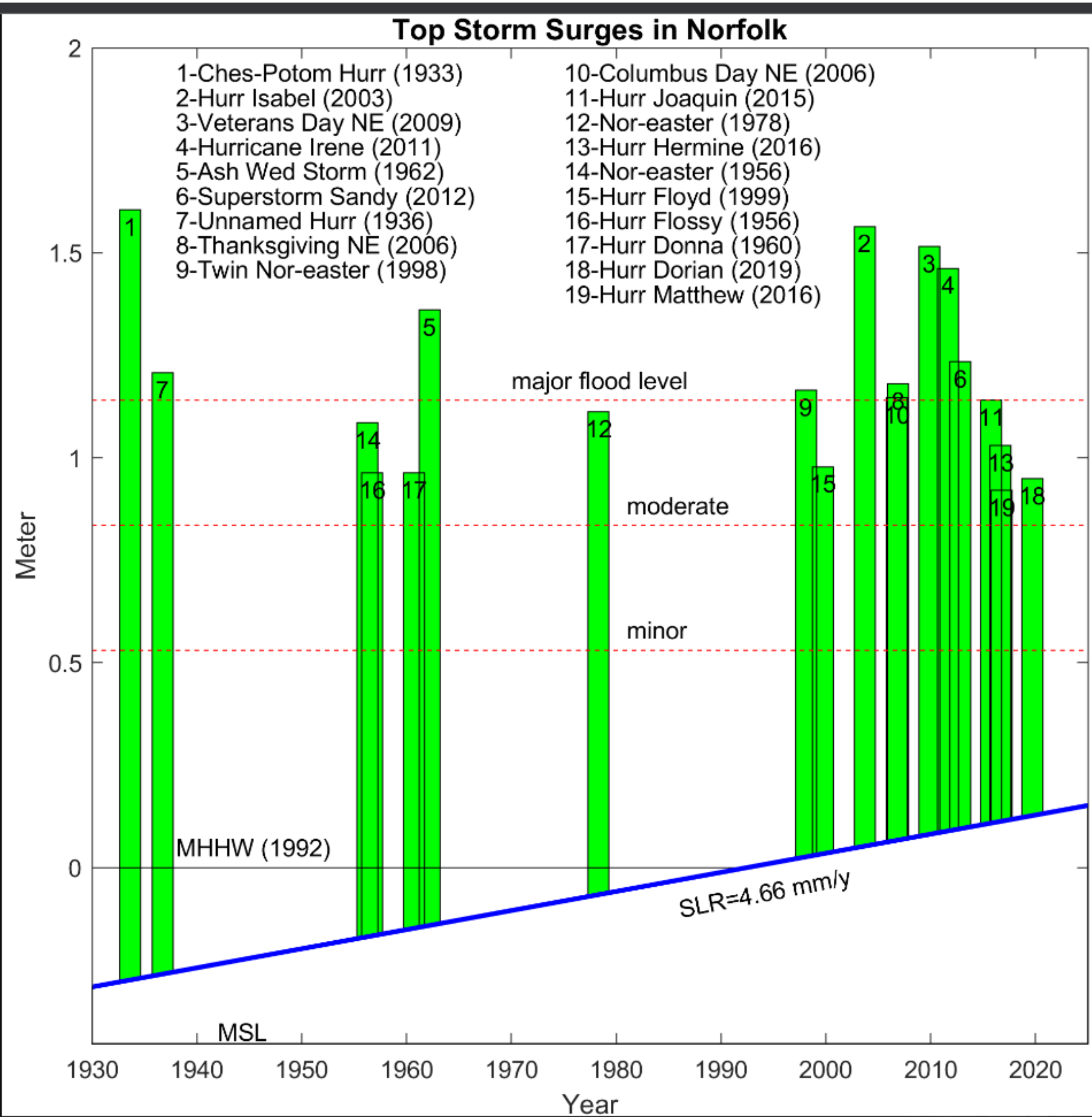
Saffir-Simpson Hurricane Scale

Category	Wind speed mph (km/h)	Storm surge ft (m)
5	≥156 (≥250)	>18 (>5.5)
4	131–155 (210–249)	13–18 (4.0–5.5)
3	111–130 (178–209)	9–12 (2.7–3.7)
2	96–110 (154–177)	6–8 (1.8–2.4)
1	74–95 (119–153)	4–5 (1.2–1.5)
Additional classifications		
Tropical storm	39–73 (63–117)	0–3 (0–0.9)
Tropical depression	0–38 (0–62)	0 (0)

famous category-5 hurricanes:

- **Dorian** (Atlantic 2019)
- Irma & Maria (2017)
- **Matthew** (Atlantic, 2016)
- Katrina, Rita & Wilma (GOM, 2005)
- Ivan (Caribbean Sea, 2004)
- **Isabel** (Atlantic coast, 2003)
- Andrew (Miami, 1992)
- Camille (GOM, 1969)

- **AFFECTED NORFOLK**



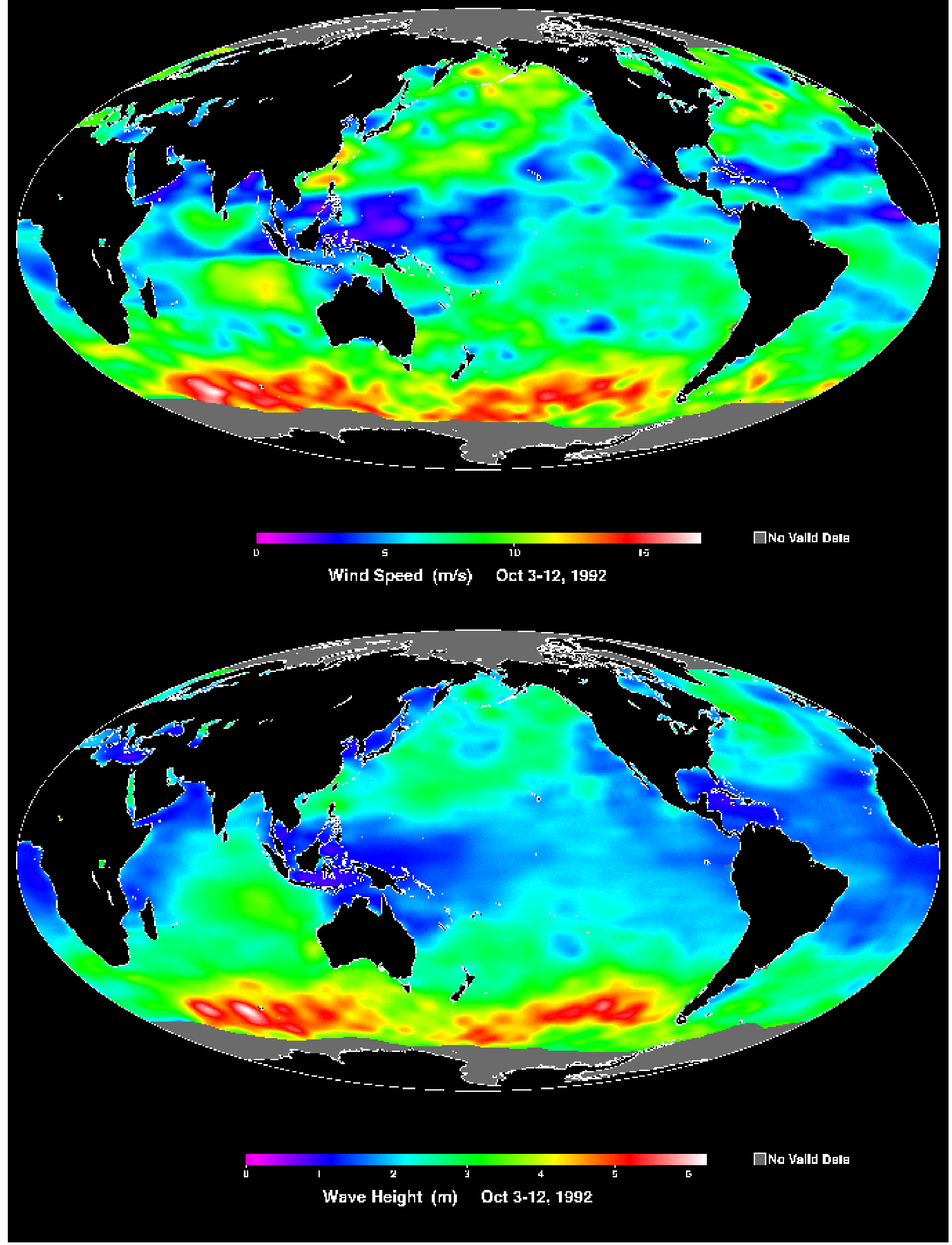
Top Storm Surges in Norfolk
1-Ches-Potom Hurr (1933)
2-Hurr Isabel (2003)
3-Veterans Day NE (2009)
4-Hurricane Irene (2011)
5-Ash Wed Storm (1962)
6-Superstorm Sandy (2012)
7-Unnamed Hurr (1936)
8-Thanksgiving NE (2006)
9-Twin Nor-easter (1998)
10-Columbus Day NE (2006)
11-Hurr Joaquin (2015)
12-Nor-easter (1978)
13-Hurr Hermine (2016)
14-Nor-easter (1956)
15-Hurr Floyd (1999)
16-Hurr Flossy (1956)
17-Hurr Donna (1960)
18-Hurr Dorian (2019)
19-Hurr Matthew (2016)
major flood level
moderate
minor
MHHW (1992)
SLR=4.66 mm/y
MSL
Meter
Year

Global observed wind speed and waves height over the oceans obtained from the Topex/Poseidon satellite altimeter in 3-12 October, 1992 (from NASA/JPL).

• There is clear a correlation between wind and waves

• Strongest winds and highest waves are found in the Southern Ocean

• Weakest winds and lower waves are found in tropical regions of the oceans (though N & S of the Equator signature of the trade winds can be seen)



Some definitions:

- **Fetch** – unobstructed distance of sea over which the wind blows
  (e.g., limited fetch in lakes and near coasts)
- **Fully developed sea** – the equilibrium state in which the size and characteristics of the waves are not changing
  (energy from wind is equal to energy dissipated by the waves)
- **Significant wave height ($H_{1/3}$)** – the average height of the highest 1/3rd of all waves occurring in a particular time period
- **Maximum wave height (H_{max})** – highest wave expected over a given period
  (e.g., H_{max}(25y) is the maximum wave expected in average over a 25-year period)
- **Swell** – long (and usually small in height) waves that are observed far away from the area in which they have been generated
  (can travel long distances with little disturbance from local wind)
- **Wave Age** – "young sea"- growing waves, near storm, short/slow waves
  "old sea" - mature waves, some time after storm, long/fast waves

(different spectrum and different roughness → different prediction models)

Long Swell off the Oregon coast generated by an offshore storm

(superimposed shorter waves are generated by local wind)

• Observed waves usually combine different wave frequencies and amplitudes, thus we need a practical definition to describe their height. The concept of **significant wave height** was developed during WW-II in attempts to predict waves.

• Suppose we count N waves from the highest to the smallest H_1, H_2, H_3,…H_N, then

$$H_{1/3} = \frac{1}{(N/3)} \sum_{n=1}^{N/3} H_n$$

• However, this is not so practical, so an easier way is to use an empirical relation to the standard deviation (SD=Root-Mean-Square)

$$H_{1/3} \approx 4 \times RMS(\eta) = 4\sqrt{\frac{1}{N} \sum_{n=1}^{N} \eta^2{}_n}$$

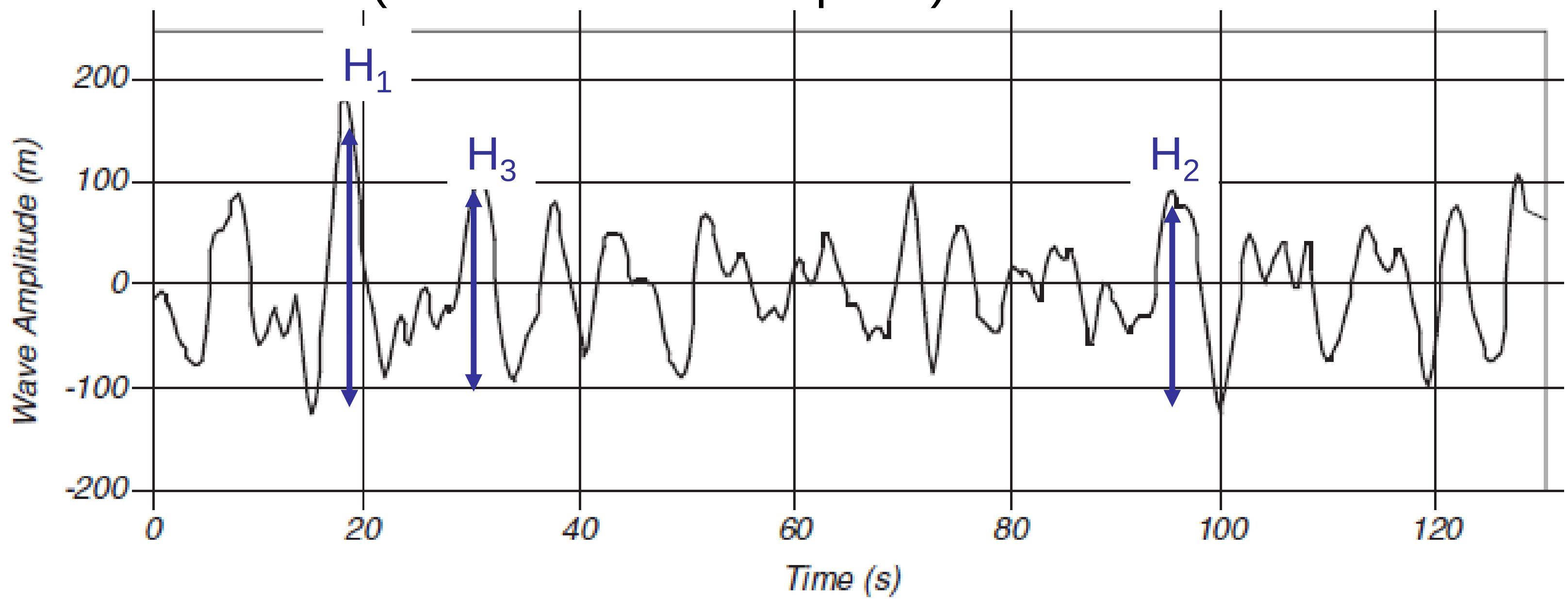
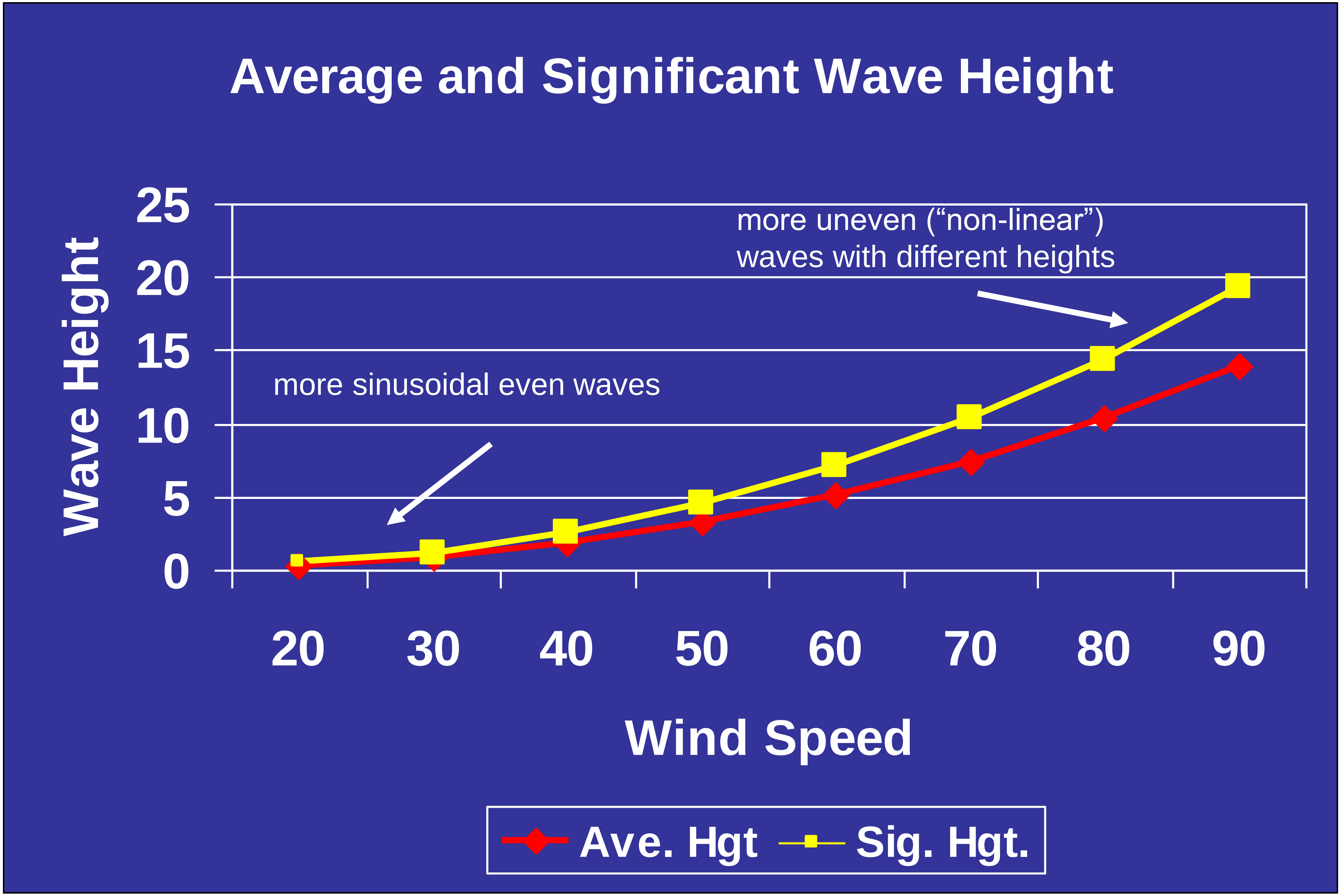


Figure 16.2 A short record of wave amplitude measured by a wave buoy in the north Atlantic.

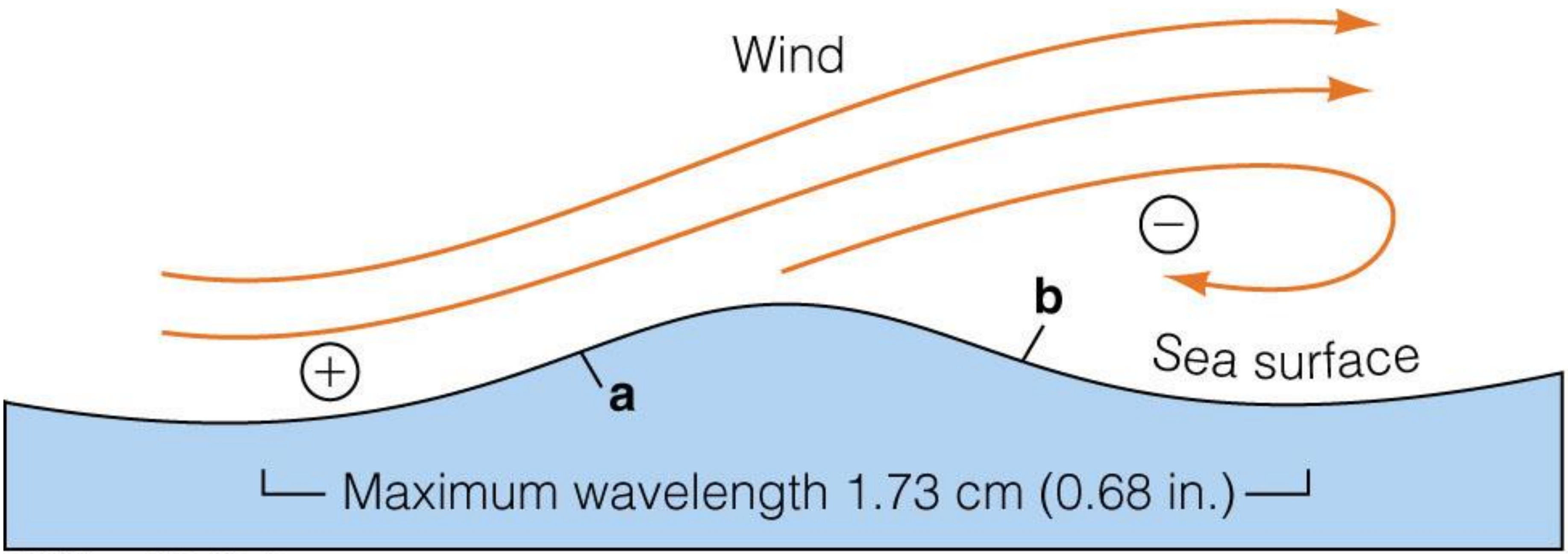

Average and Significant Wave Height
Wave Height
25
20
15
10
5
0
more uneven ("non-linear")
waves with different heights
more sinusoidal even waves
20
30
40
50
60
70
80
90
Wind Speed
Ave. Hgt
Sig. Hgt.

How do wind-waves develop?

Wind starts blowing over a smooth sea surface and generates small structures that increase the sea roughness

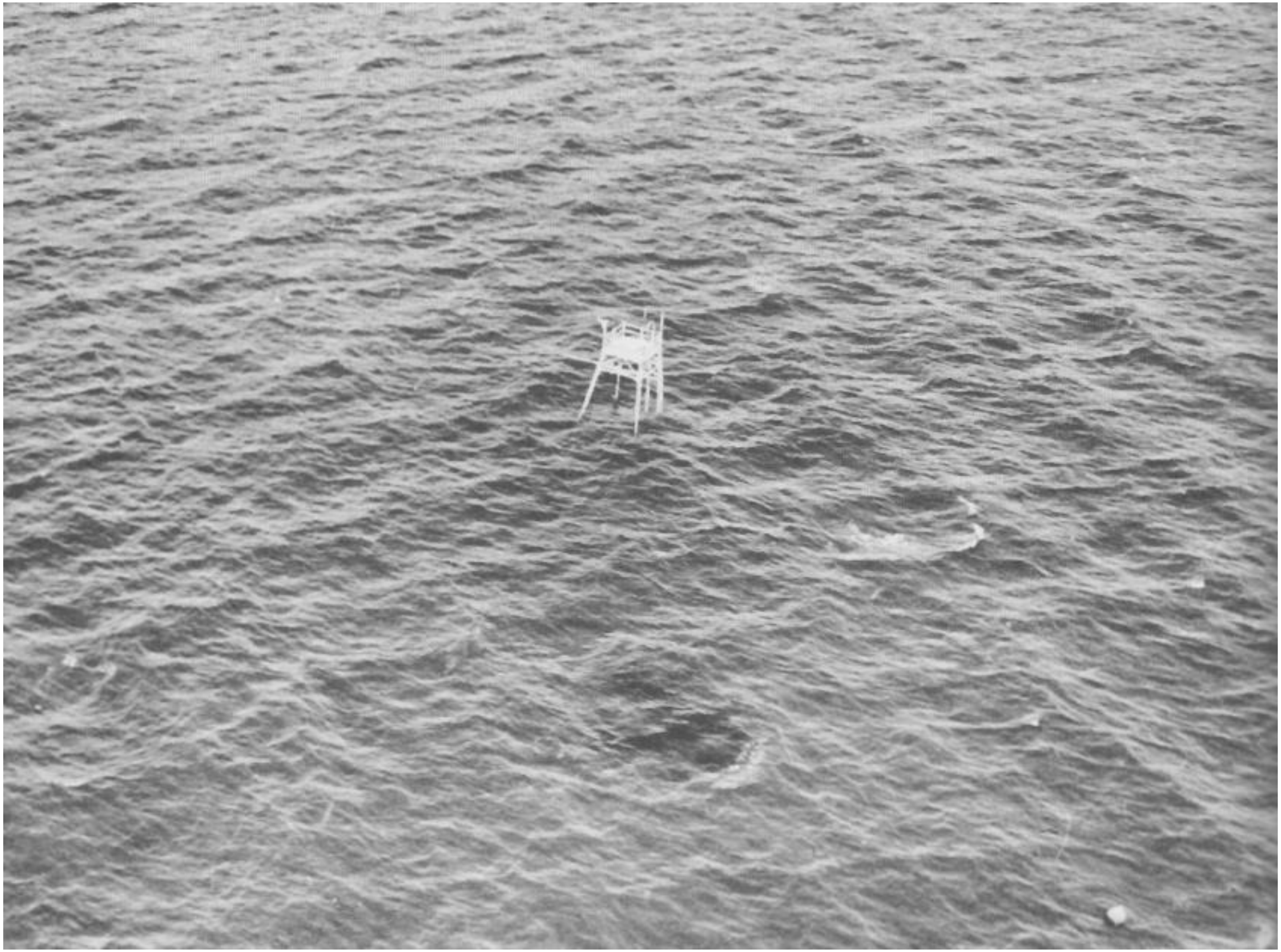
(from Kinsman, 1965)

The development of Capillary waves is important at this stage, disturbing the smooth sea surface and creating turbulence in the wind field above the ocean

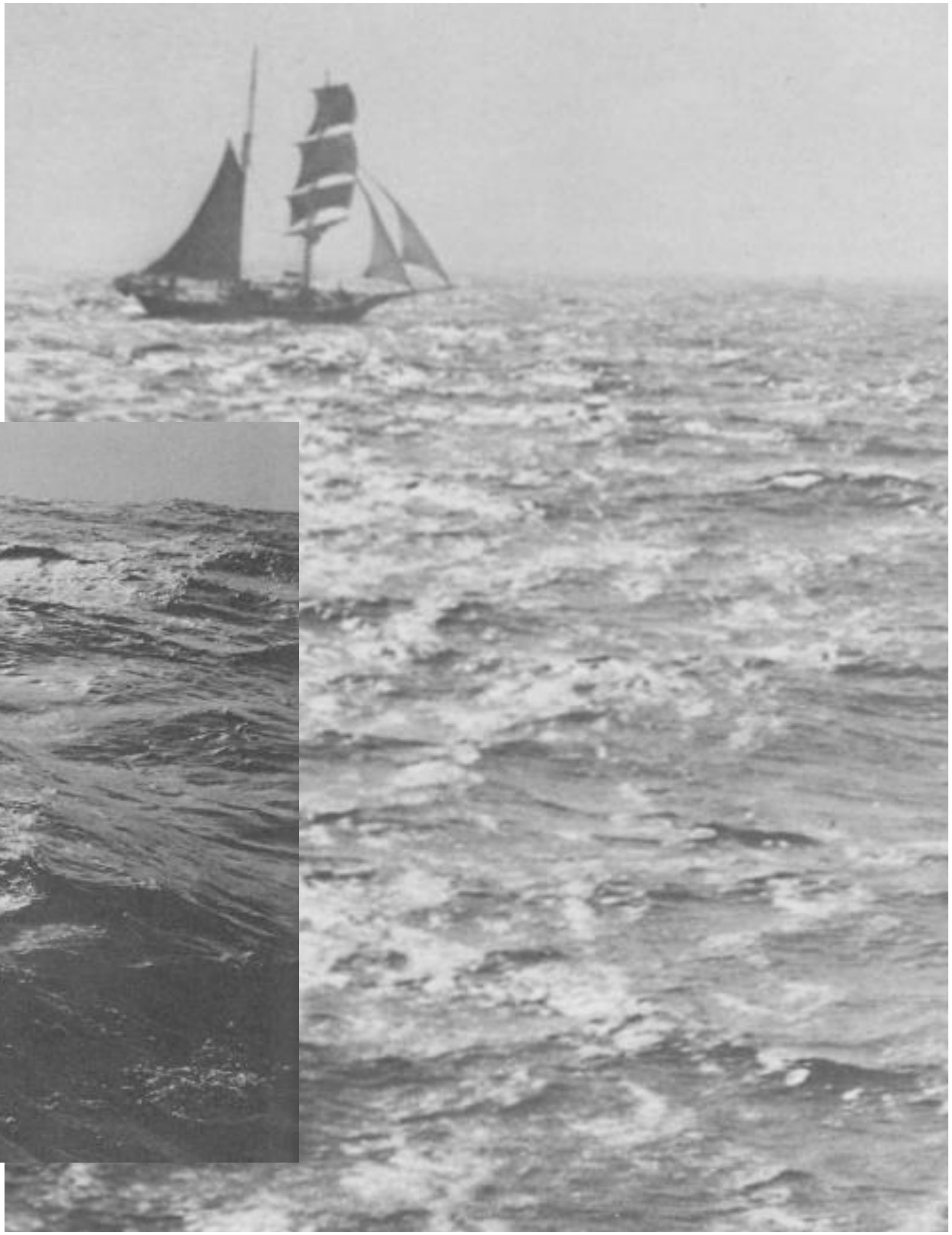


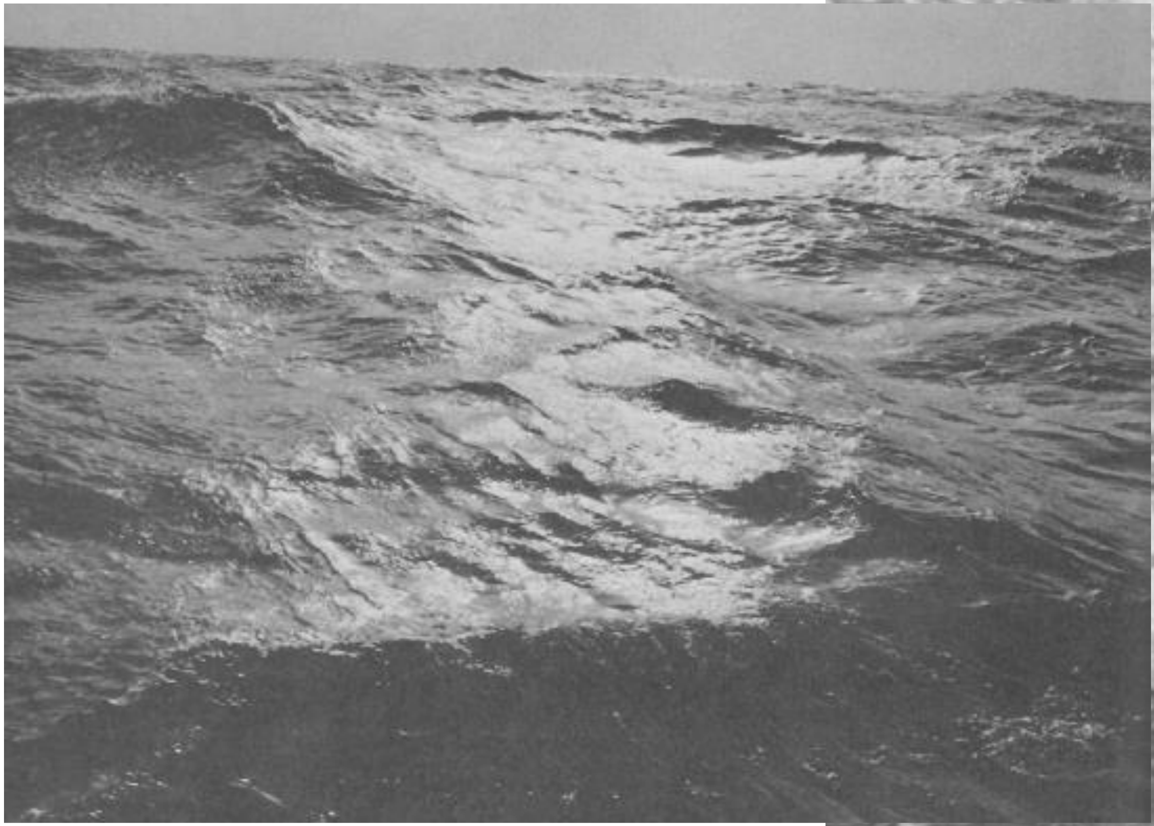
In early stages of the growing sea there are some regular wave patterns and some irregular waves

(difficult to describe mathematically the process of a growing sea)



The complex form of a developed sea includes different waves (“old”/long and “young”/short) superimposed.





When waves become too steep they will start breaking

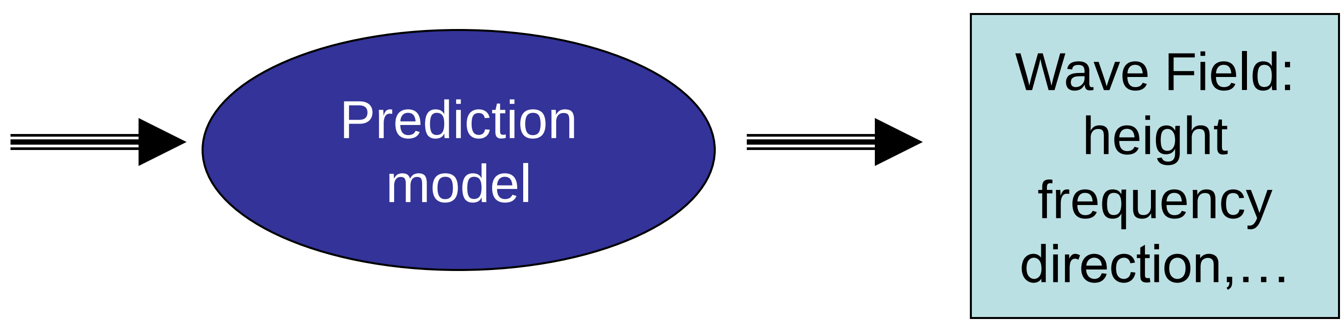
(the mathematical description of breaking waves is non-linear and more complex than propagating waves)

Stages of sea development:

- A wind starts to blow over a calm sea, increasing to a gale force and continues to blow for a considerable period
- Waves start to develop when wind speed reaches 1 m/s
- Small steep waves form as the wind increases
- Waves continue to grow until their phase speed is about 1/3 of the wind speed
- Waves growth slows considerably or stop due to:
  - Energy dissipation by friction
  - Waves break when becoming steep (“white-capping”)
  - Wind energy converted to ocean currents instead of waves

Parameters controlling the wave field:

- **Wind speed**
- **Wind duration**
- **Fetch**



Theoretical models, empirical models and computer (numerical) models

Generation of deep water waves by the wind:

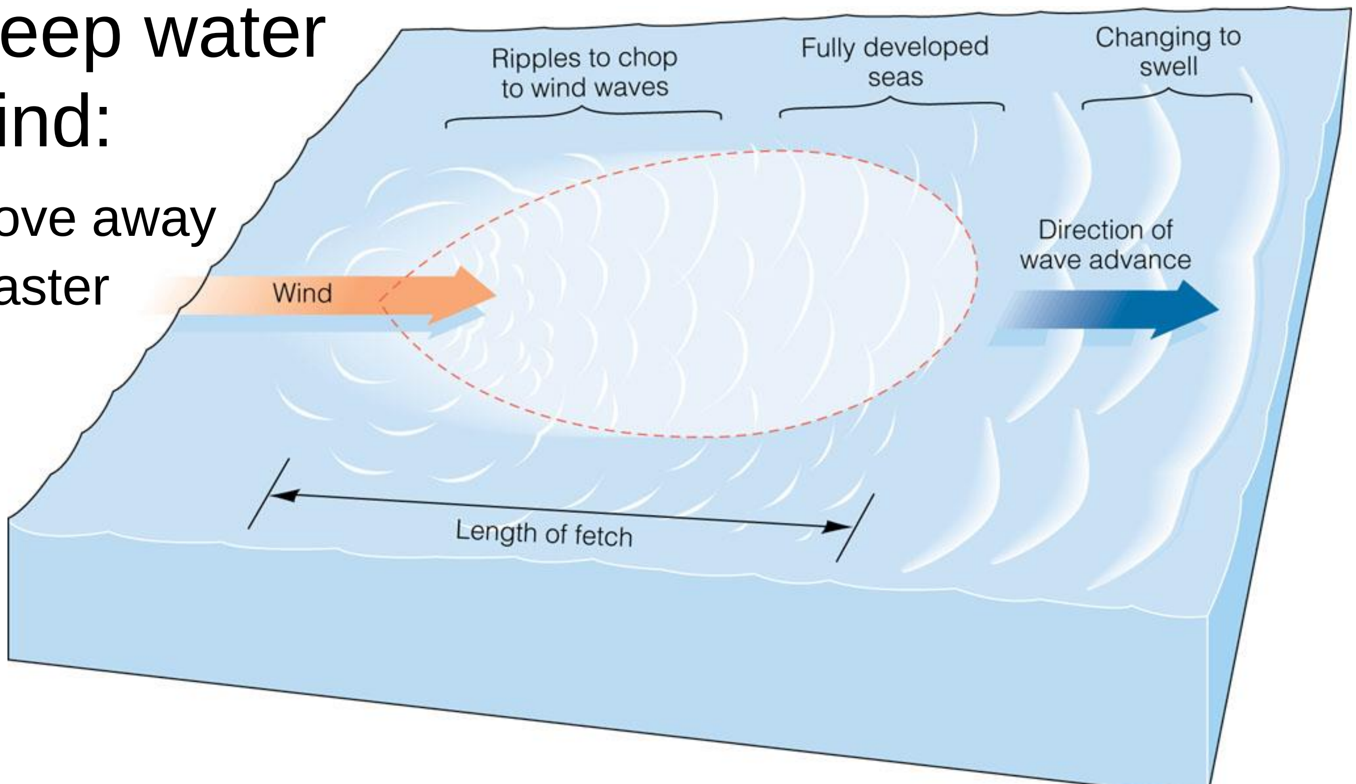
- longer waves will move away from the storm area faster than shorter waves.

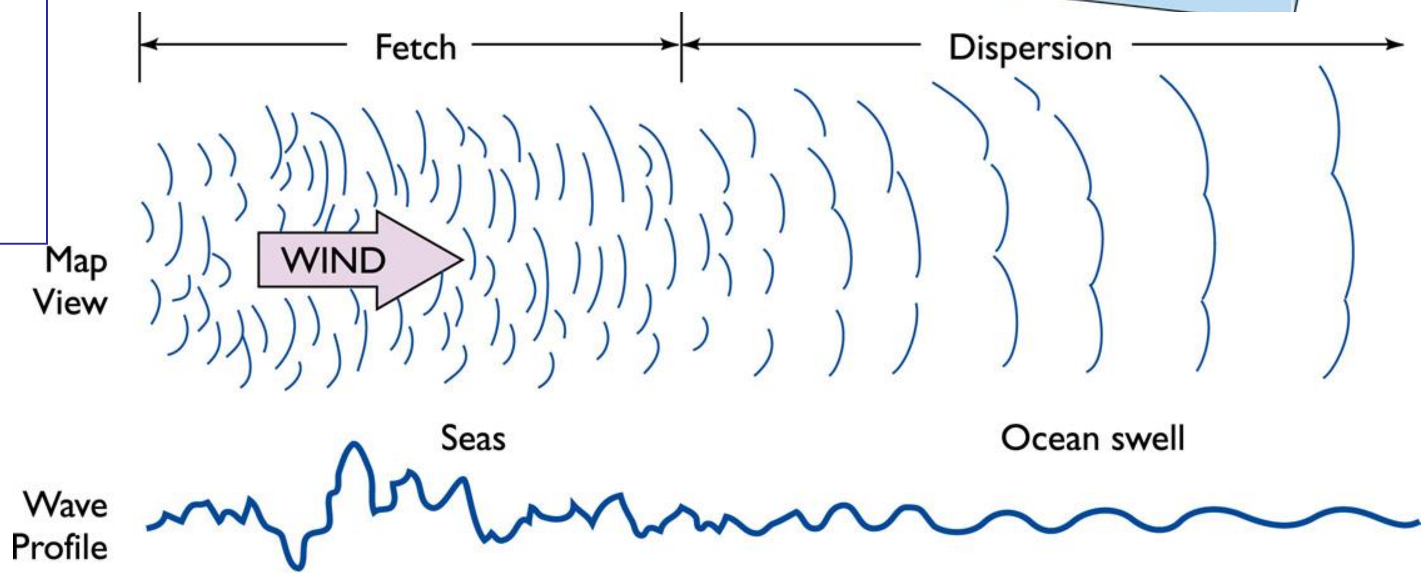
Deep-water wave velocity is:

$$c = \sqrt{\frac{g}{k}} = \sqrt{\frac{g}{2\pi}\lambda}$$

or

$$c = \frac{g}{2\pi}T$$





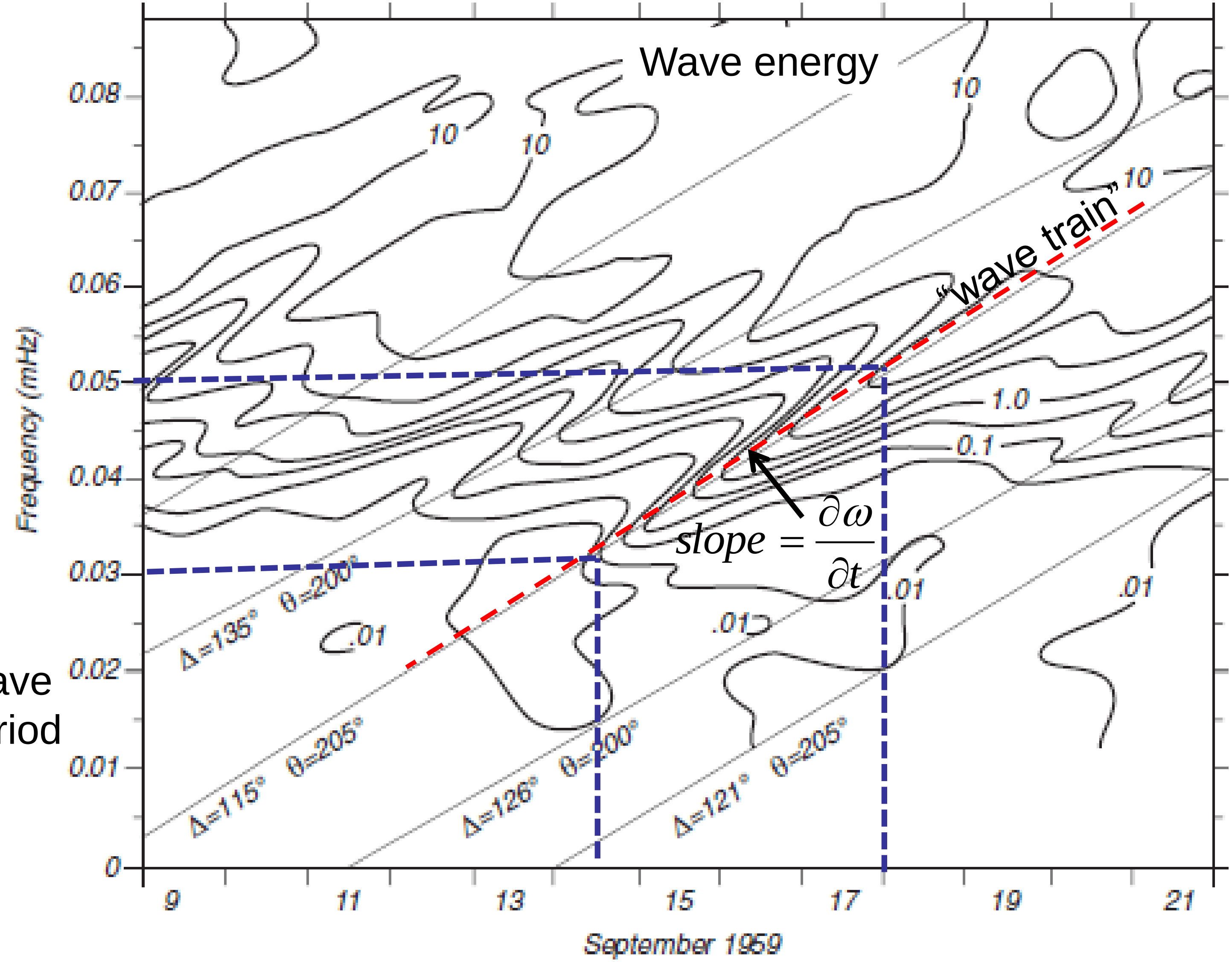
Observations of waves off the California coast (Munk, 1963)

T=20sec

T=33sec

wave period

Note: longer waves arrive first



One can calculate the distance to the storm (Δ) from the slope of the plot ∂ω/∂t

(note: energy propagates at group velocity)

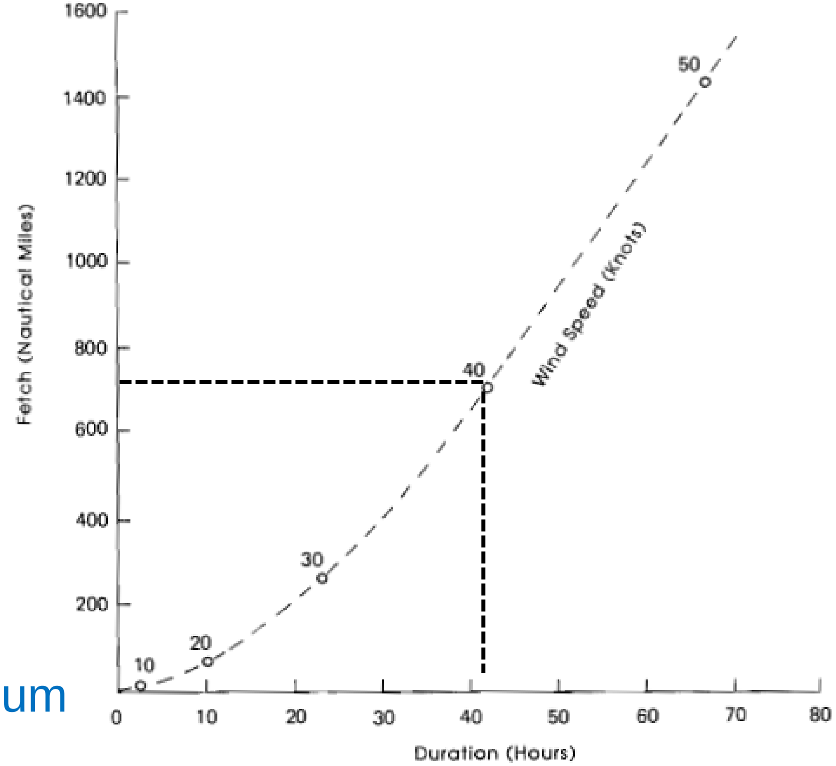
$$c_g = \frac{g}{4\pi}T = \frac{g}{4\pi}\left(\frac{2\pi}{\omega}\right) = \frac{g}{2}\left(\frac{1}{\omega}\right)$$

$$\Delta = c_g \Delta t = \frac{g}{2}\left(\frac{1}{\Delta\omega}\right)\Delta t \rightarrow \frac{\Delta\omega}{\Delta t} = \frac{g}{2}\left(\frac{1}{\Delta}\right)$$

3 factors provide the conditions for fully developed sea:

(1) wind speed
(2) wind duration
(3) fetch

these factors can provide an estimate of the waves at **fully developed sea**



However, if one of the 3 factors is not satisfied, then waves will not be developed to their maximum potential (example: strong wind, short duration)

Wind Conditions			Wave Size		
Wind Speed in One Direction	Fetch	Wind Duration	Average Height	Average Wavelength	Average Peri
19 km/hr (12 mi/hr)	19 km (12 mi)	2 hr	0.27 m (0.9 ft)	8.5 m (28 ft)	3.0 sec
37 km/hr (23 mi/hr)	139 km (86 mi)	10 hr	1.5 m (4.9 ft)	33.8 m (111 ft)	5.7 sec
56 km/hr (35 mi/hr)	518 km (322 mi)	23 hr	4.1 m (13.6 ft)	76.5 m (251 ft)	8.6 sec
74 km/hr (46 mi/hr)	1,313 km (816 mi)	42 hr	8.5 m (27.9 ft)	136 m (446 ft)	11.4 sec
92 km/hr (58 mi/hr)	2,627 km (1,633 mi)	69 hr	14.8 m (48.7 ft)	212.2 m (696 ft)	14.3 sec

Increased Wind Speed causes waves with:

- increased height

- increased wavelength

- increased period

Why does wavelength grow faster than wave period (which has linear relation with wind speed)?

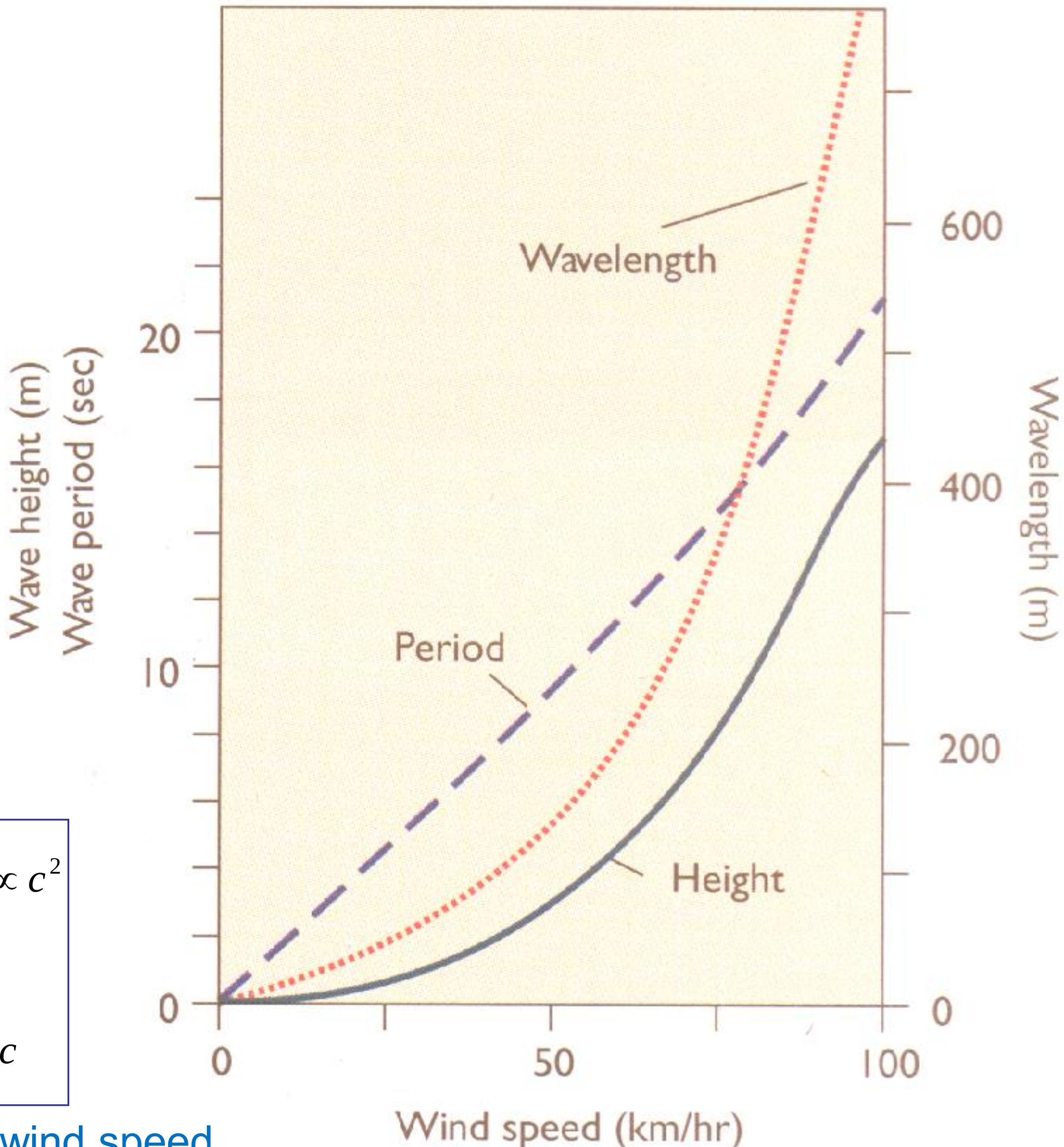
Possible explanation:

$$c = \sqrt{\frac{g}{2\pi}\lambda} \rightarrow \lambda \propto c^2$$

but

$$c = \frac{g}{2\pi}T \rightarrow T \propto c$$

So if wave speed is related to wind speed…



Wave Spectra:

wave energy as a function of frequency

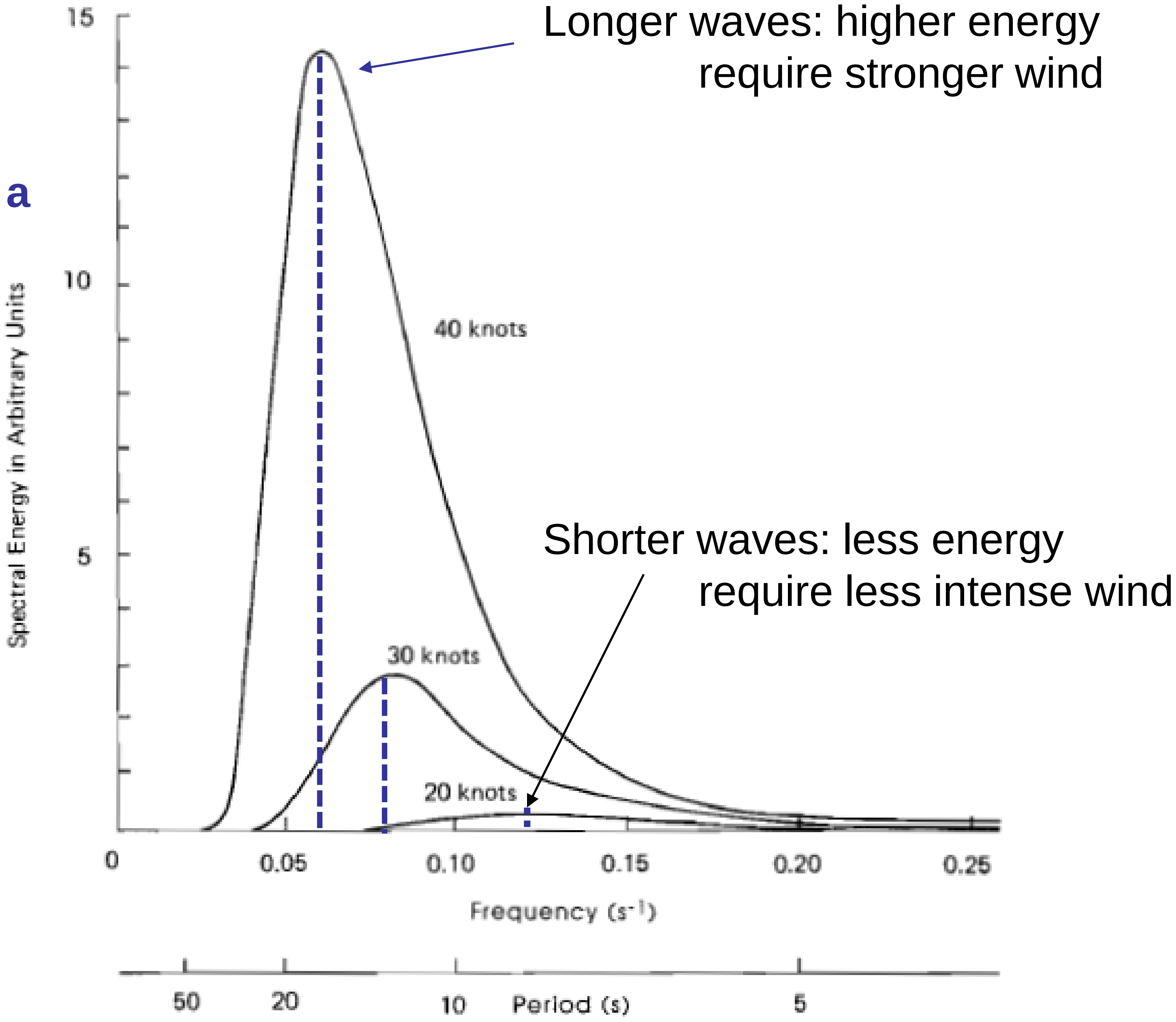


Figure 9.11 Idealized wave spectra for a fully developed sea for winds of 20, 30, and 40 knots.

Wind-Waves Models:
Waves physics, evolution and prediction

~ What is the mechanism of wind waves growth and decay?

~ Can we describe the wind-wave physics in mathematical equations?

~ Can we predict the exact sea state from wind data?

Unfortunately, the answers to these questions are not fully resolved even today…

… so, we will review some of the attempts over the past century to understand wind-wave interaction and developed models to predict waves

Progress in wind-waves physics and prediction models

- Empirical relations between wind and waves, observations of storm and estimated swell propagation since the 1920s.

• Jeffreys (1925)

• Sverdrup-Munk (1945)

- Air-sea interaction studies and spectral models:

• Phillips (1957) & Milles (1960), Pierson-Neuman (1957, 1961), Pierson-Moskowitz (1964)

- Numerical (computer) wave prediction models:

• First generation wave models (no non-linear interactions) - 1970s

• Second generation wave models (parameterization of interactions) – 1980s

• Third generation wave models (solving (almost) all physics) – 1990s
[WAM, SWAN, WAVEWATCH, etc.]

• Coupled operational wave-current models and data assimilation- 2000-now

Note that some of the early models were not totally correct… but each shed some light on better understanding wave physics.

• **Jeffreys (1925)** “sheltering” model suggests that pressure differences between the front and the back side of the wave drives the wave propagation in the direction of the wind. This effect, while partly correct for steep enough waves, can not explain the early stages of growing seas and contradicts lab experiments.

•The criterion for wave growth according to that model:

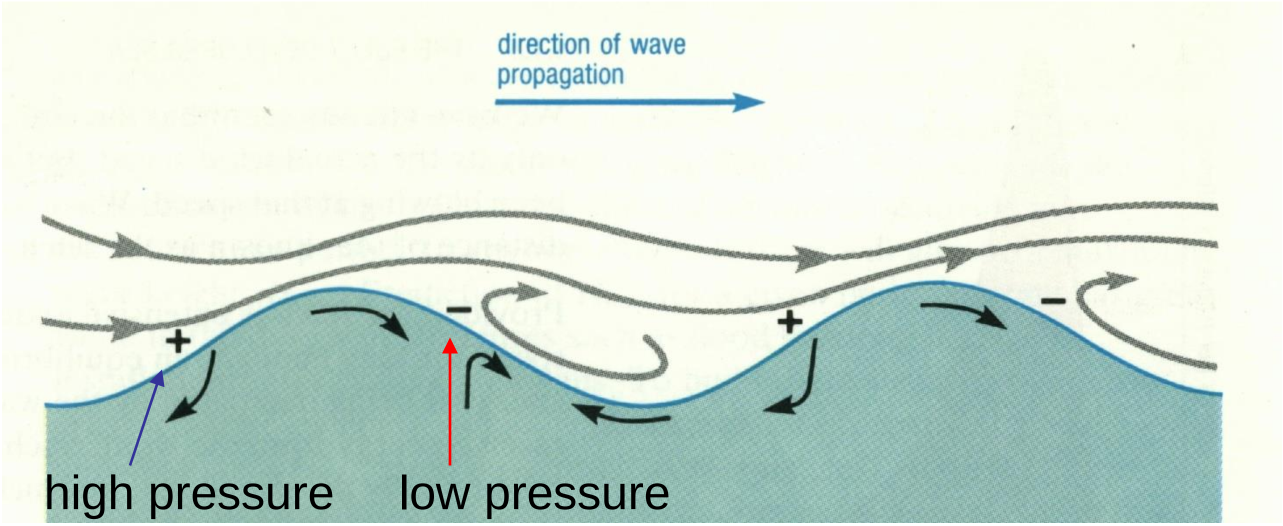
$s\rho_{air}(U-c)^2U>4\mu_{air}g$

ρ, μ= density & visc. of air

s=“sheltering coef.”

U=wind speed

c=wave speed



• **Sverdrup and Munk (1943)** developed a theory of waves aimed to find practical techniques for forecasting wind waves for Navy operations during WW-II efforts.

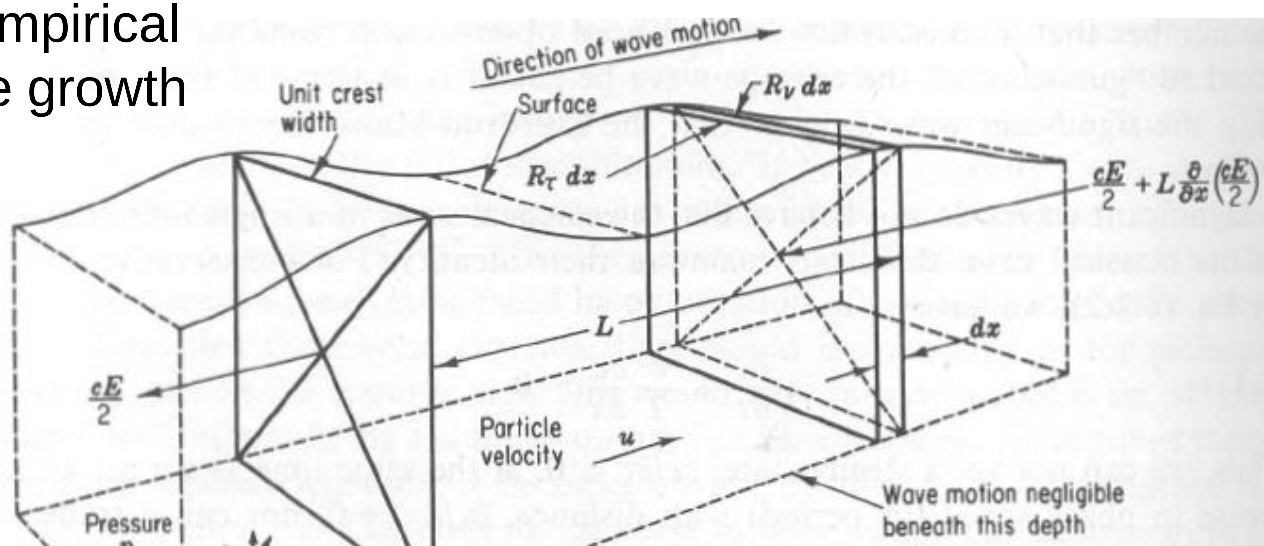
• They realized that wave theory must be statistically-based

• They introduced the concept of "**significant wave height**" and tried to predict it from air and sea data (even though there is no such "a wave").

• Their assumption: a constant wind blowing only inside a "box storm".

• They developed semi-empirical equations to predict wave growth and decay.

• While their theory generally agreed with observations available at the time, it was not completely correct and did not explain the actual mechanism of energy transfer from wind to waves (energy to currents neglected).



tangential & pressure wind stress

Energy budget eq. $$\frac{d(EL)}{dt} = \left[R_\tau \pm R_\nu + \frac{\partial}{\partial x}\left(c\,\frac{E}{2}\right)\right] L$$

Fig. 6.4-1. Energy changes for an individual wave of length L moving with phase speed c. [After Sverdrup and Munk (1947)]

Sverdrup-Munk **wave growth theory**: developed relations between wave steepness, $\delta = H/L$, wave "age", $\beta = c/U$ (U=wind), fetch and wind speed and duration

If we now *assume* that the wave steepness δ is a function of the wave age β *only*, we can write either

$$\frac{d\delta}{dx} = \frac{d\delta}{d\beta}\frac{d\beta}{dx} \quad \text{or} \quad \frac{d\delta}{dt} = \frac{d\delta}{d\beta}\frac{d\beta}{dt} .$$

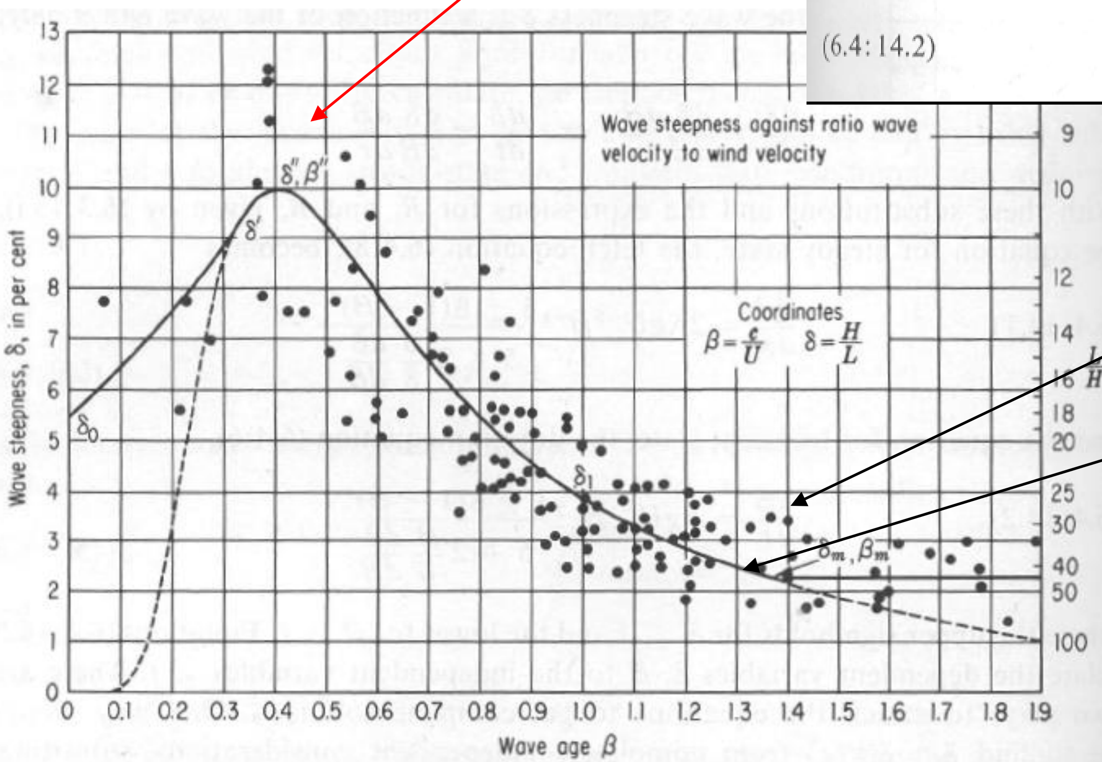
With these substitutions and the expressions for R_τ and R_ν given by (6.3:15.i), the equation for steady state, the fetch equation (6.4:8), becomes

$$\frac{d\beta}{dx} = 2\mathrm{A}gU^{-2}\beta^{-3}\frac{1 \pm \mathrm{B}(1-\beta)^2}{5 + 2\dfrac{\beta}{\delta}\dfrac{d\delta}{d\beta}} \tag{6.4:14.1}$$

and the equation for transient state, the duration equation (6.4:6),

$$\frac{d\beta}{dt} = \mathrm{A}gU^{-1}\beta^{-2}\frac{1 \pm \mathrm{B}(1-\beta)^2}{5 + 2\dfrac{\beta}{\delta}\dfrac{d\delta}{d\beta}} , \tag{6.4:14.2}$$

Maximum wave steepness



observations

prediction

Fig. 6.4-2. Wave steepness as a function of wave age. The plotted points are observed values. The assumed functional relation is given by the solid line. [After Sverdrup and Munk (1947)]

The theory tries to predict how waves grow with time given wind duration, speed and fetch.

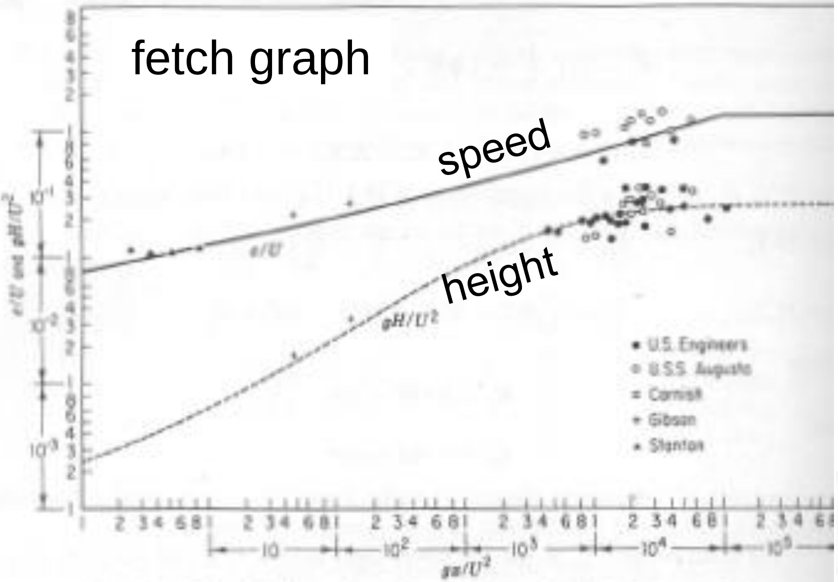


Fig. 6.4-3. Wave height and speed as functions of fetch. The theoretical functional dependence is shown by the curves, the supporting data by the plotted points. [From Sverdrup and Munk (1947)]

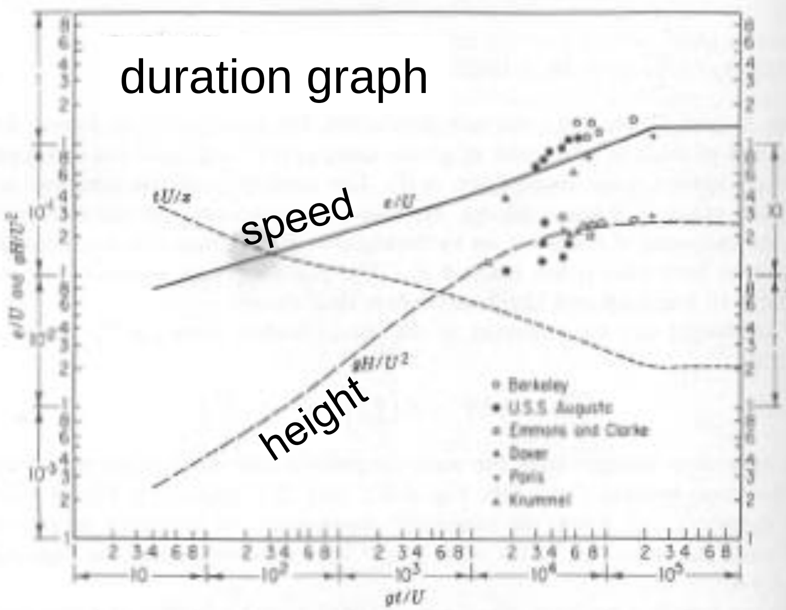


Fig. 6.4-4. Wave height and speed as functions of duration. The theoretical functional dependence is shown by the curves, the supporting data by the plotted points. [From Sverdrup and Munk (1947)]

Growth of wave height with time

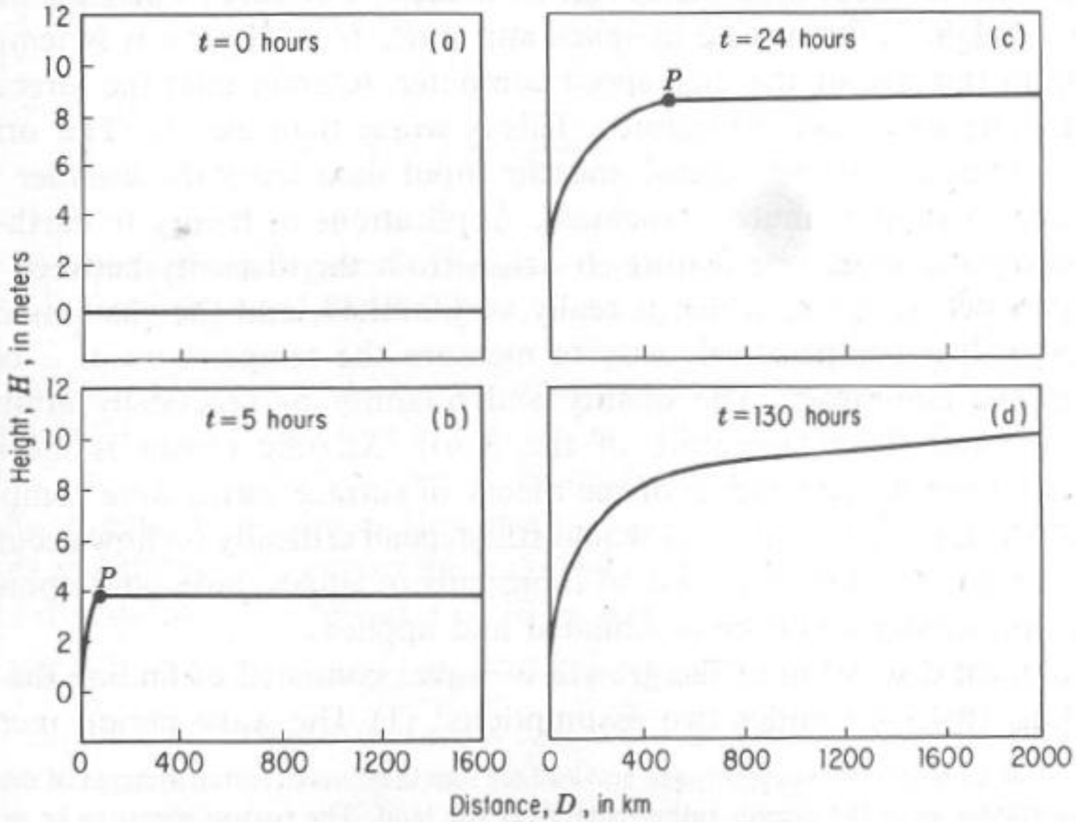


Fig. 6.4-5. Growth of wave height with time and distance from the upwind edge of a fetch. [From Sverdrup and Munk (1947)]

Sverdrup-Munk **wave decay theory**: (swell) waves have left the generation area and move into a region of a calm sea. They loose energy through friction with the air (which now does not move in the direction of the waves).

Energy loss: $-R_\nu=(1/8)s\rho_{air}(gH)^2/c$

s="sheltering coeff."
ρ= density of air
H= wave height
c= wave speed

T_D= wave period at the decay distance D
T_F= wave period at the edge of the fetch area
t_D= time wave traveled distance D

Note that only few data points exist to test the accuracy of the model

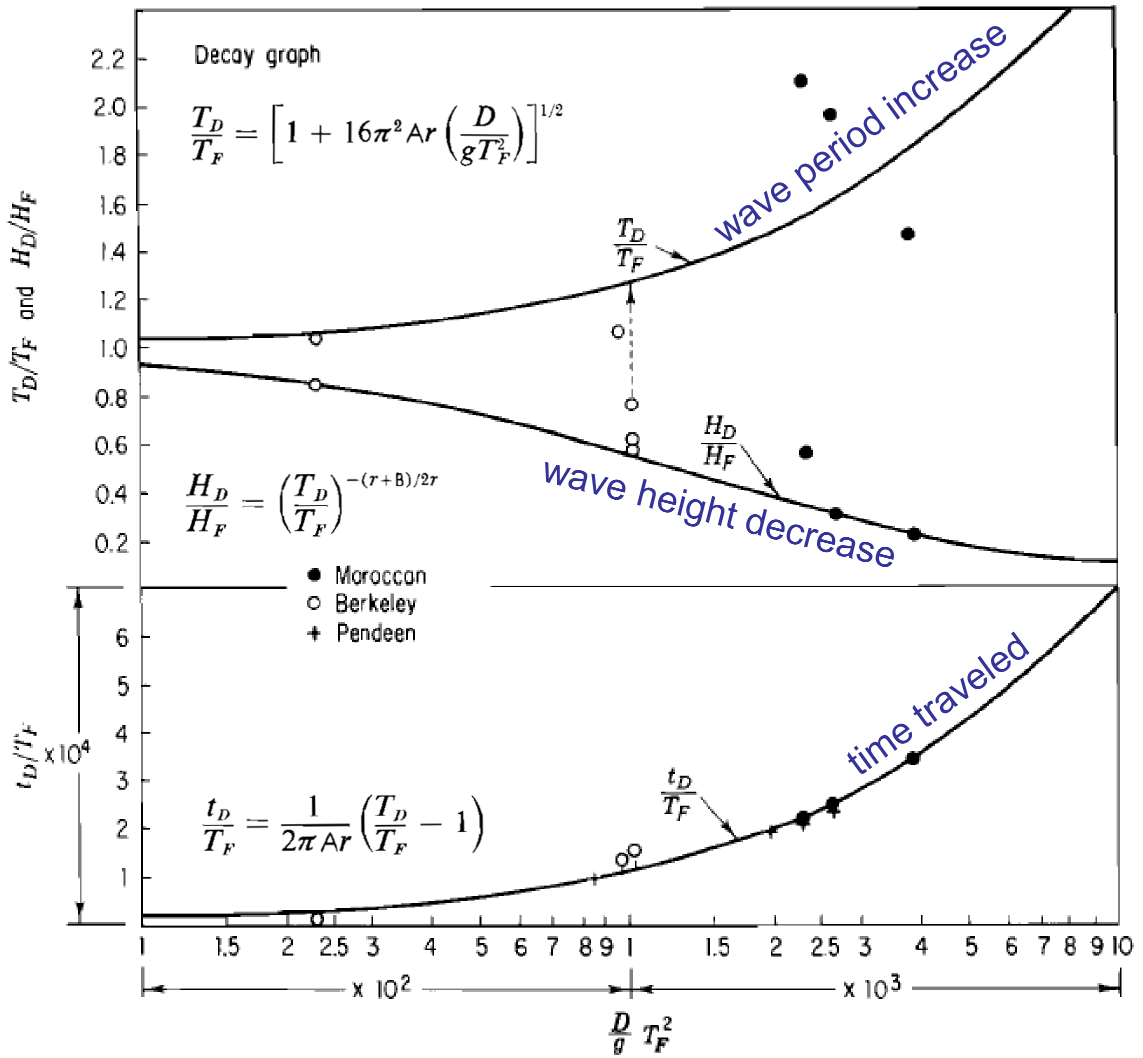


Fig. 6.5-1. Period and height of swell as functions of decay distance and travel time. The theoretical functional dependence is shown by the curves, the supporting data by the plotted points. [From Sverdrup and Munk (1947)]

Summary so far:

• **Jeffreys (1925)** provides the idea of pressure differences- a grain of truth, but not fully supported by observations.

• **Sverdrup and Munk (1943)** model was based on empirical relations, but not a real theory of wind-wave dynamics. (“fit” the theory with data)

• **Ursell (1956)** summarized the state of our knowledge of wave generation by wind: *“… nothing very satisfying*”

<u>Some breakthrough in understanding:</u>

• **Phillips’ (1957) resonant model**: pressure fluctuations of the turbulent wind.

• **Miles (1960) shear flow model**.

<u>Some breakthrough in wave prediction:</u>

• **Pierson-Neuman (1957, 1961)** and **Pierson-Moskowitz (1964)** developed **spectrum model**: combine stochastic approach, power spectrum and observations to predict the behavior of surface wind waves.

• Numerical wave models (Komen et al., 1994, and others)

Phillips' (1957) "resonant" model:

First model to consider the air-sea wave interaction

- pressure fluctuations of the turbulent wind creates roughness of the sea (Capillary waves).

- As the waves grow, so is the reaction of wind to the waves (the "Jeffreys' sheltering effect"), causing a constantly changing balance (wave-wind feedback).

- The mathematics are quite complex, so we will not developed here the full set of equations, just discuss the basic concept and results.

Phillips' (1957) "resonant" model

Formulation of the wind waves problem for deep waters

Laplace's eq for the velocity potential:

$$\nabla^2 \phi = 0$$

Reminder: $u=\phi_x$, $v=\phi_y$, $w=\phi_z$

With BCs for the deep ocean:

$$\phi\big|_{z\to-\infty} \to 0$$

And for the surface:

$$\frac{\partial \eta}{\partial t} = \frac{\partial \phi}{\partial z}\bigg|_{z=0}$$

$$\frac{\partial \phi}{\partial t} + g\eta = \boxed{\frac{p}{\rho}} + \boxed{\frac{\tau}{\rho}}\nabla^2 \eta\bigg|_{z=0}$$

Note that unlike the Bernoulli's eq with constant pressure now the forcing for the surface elevation, $\eta(x,y,t)$, is the fluctuations in surface pressure, $p(x,y,t)$; we also include surface tension, τ.

Representing the variables in terms of their Fourier transform and substituting into the eqs.

$$\eta(x,t) = \int B(k,t)e^{ikx}dk$$

$$p(x,t) = \int \varpi(k,t)e^{ikx}dk$$

$$\phi(x,z,t) = \int (1/k)B'(k,t)e^{kz}e^{ikx}dk$$

ϖ=spectrum of air pressure fluctuations

B=spectrum of sea surface pressure

So we get an equation that relate the surface elevation spectrum (B) to the forcing air pressure fluctuations (ϖ):

$$B''(k,t) + \omega^2 B(k,t) = (k/\rho)\varpi(k,t)$$

ω is the frequency of the waves when both gravity and surface tension are important

$$\omega^2 = gk + \tau k^3/\rho$$

The full solution is complex, but it has the following characteristics:

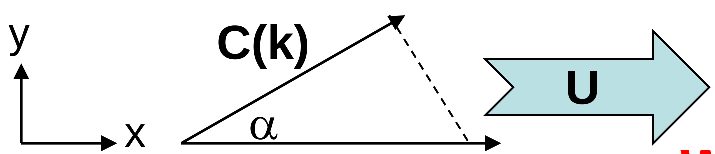
• The wave energy growth is a function of the pressure spectrum and proportional to:

$$dE(k) \sim k^2 \, func\left[\frac{1}{(\omega_{air} - \omega_{water})}\right]$$

• higher wave number (shorter waves) are excited more easily by smaller wind fluctuations- agree with observations that wave growth start from small waves (~1 cm), or capillary waves.

• there is a “resonance” when $\omega_{air} \sim \omega_{water}$ with max wave growth

• the resonance condition for the wind U is: (wind component in wave direction forces waves with wavenumber k)



$$U \cos\alpha = \sqrt{\frac{g}{k} + \frac{\tau k}{\rho}} = c(k)$$

wave speed ∝ wind speed

Some other useful results:

- Minimum wave speed

$$c_{\min} = \left[\frac{4g\tau}{\rho}\right]^{1/4}$$

- Critical wavelength

$$L_{cr} = 2\pi\left[\frac{\tau}{\rho g}\right]^{1/2} \rightarrow k_{cr} = \left[\frac{\rho g}{\tau}\right]^{1/2}$$

- Critical angle for resonance waves

(wind direction and wave propagation direction are not necessarily the same)

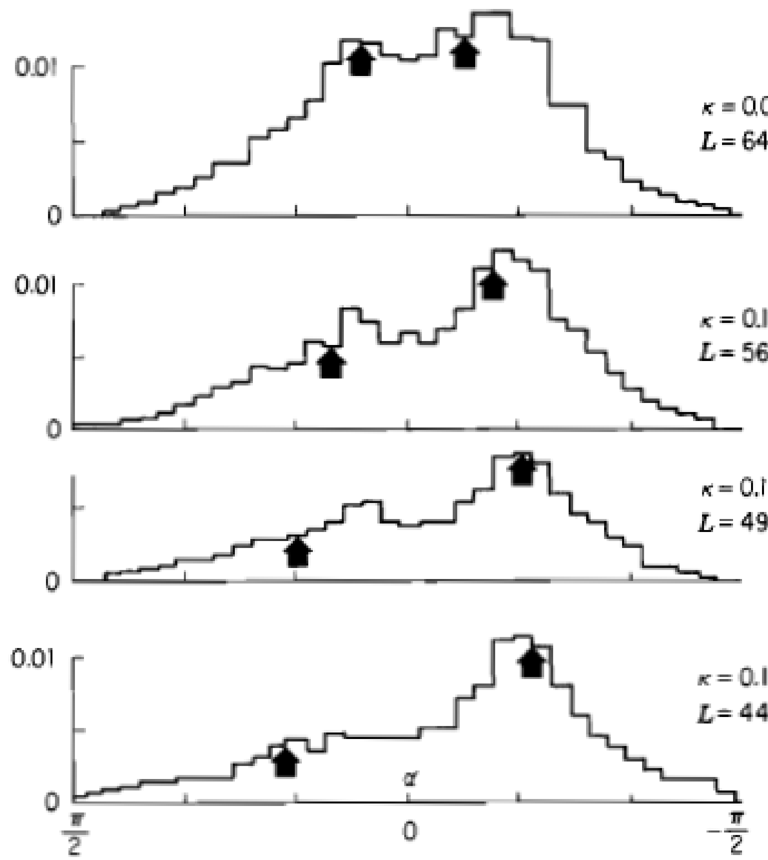
$$\alpha_{cr} = \cos^{-1}\left(\frac{c_{\min}}{U}\right) = \cos^{-1}\left[\frac{4g\tau}{\rho U^4}\right]^{1/4}$$

C_{min}~ 23 cm/s; L_{cr}~ 1.7 cm (Capillary waves theory).

However, the observed U_{min} to generate resonant waves measured as ~40-1200 cm/s… not exactly fit theory.

Prediction of wave dependency on wind: $$\frac{k}{k_{cr}} = U^2\left(\frac{\rho}{4g\tau}\right)^{1/2} \pm \left[U^4\left(\frac{\rho}{4g\tau}\right) - 1\right]^{1/2}$$

Observed wave spectrum as a function of wave number and wind angle- the angle pf the critical resonance may not always agree with observations



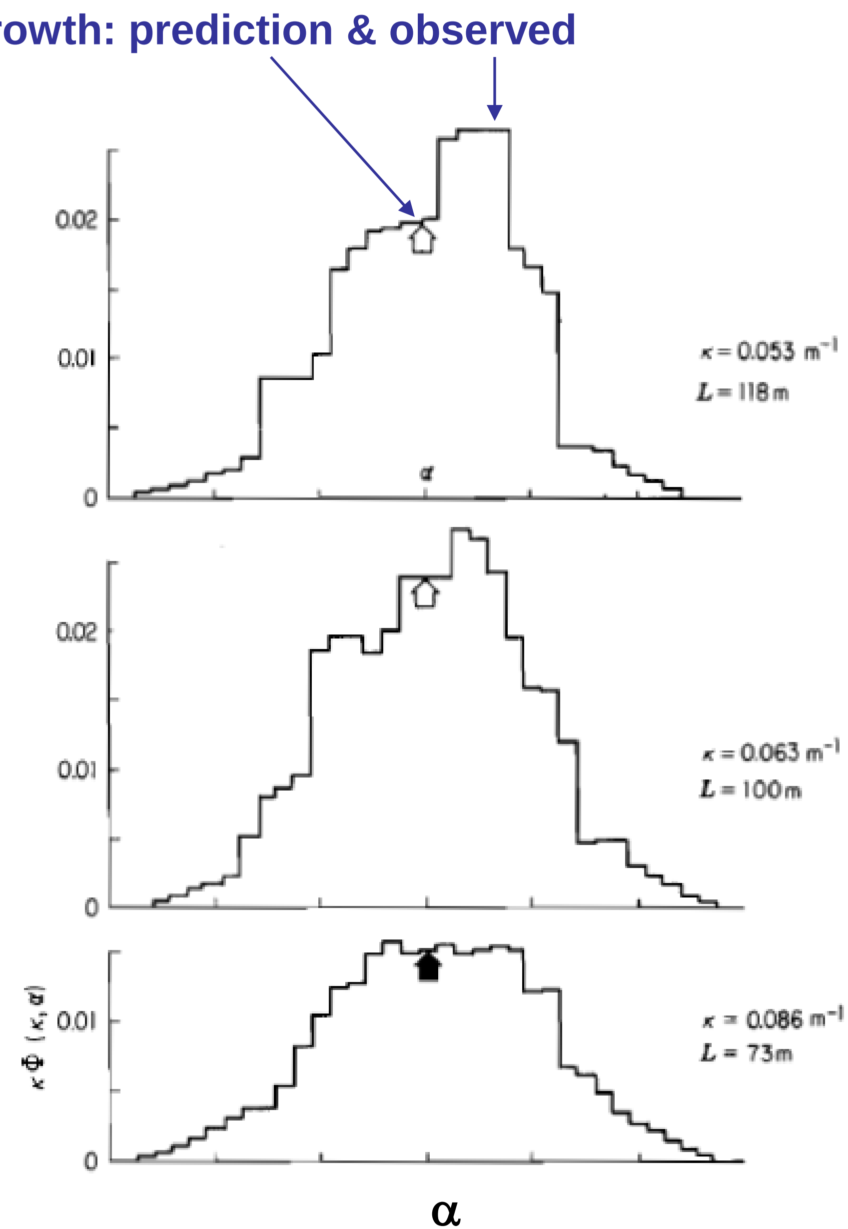
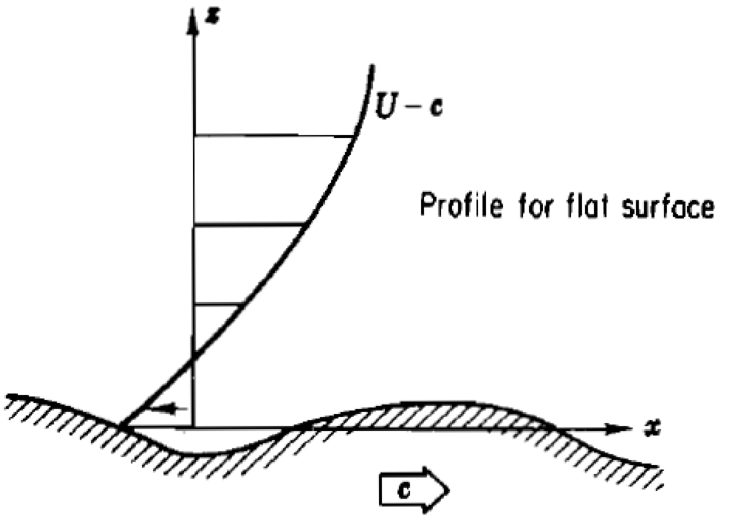


Fig. 12.1-4. Sections of the weighted instantaneous wave spectrum $\kappa\Phi(\kappa, \alpha)$ for fixed wave numbers κ, determined from the SWOP data. [From Phillips (1958b)]

Next class:



-Miles' (1957) Shear-Flow model
(air-sea interaction model)

-Pierson (1957, 1964) spectral wave models
(statistical models)

- Directional wave models
- Wave charts prediction
(practical wave prediction methods)